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EDITED BY

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EDITORIAL.

Definitions.

To define, as the word is ordinarily used in elementary mathematics, is to state precisely the meaning of, or to describe the nature and certain determining properties of the idea defined. To be a good definition the description must be in terms of simpler ideas than the thing defined, and must be sufficiently comprehensive to include every member of its class. It must be *inclusive,* in that it includes all of the class defined, and *exclusive,* in that it excludes all others. It is therefore a statement or series of statements of which the converse is also true, otherwise it fails in this exclusiveness. It would be true to say that a horse is a quadruped, but it would not define a horse, as all quadrupeds are not horses.

Definitions have a twofold purpose; first, to give the exact meaning of the idea defined in an intelligible form, and second, to give in concise form the determining properties, so that they may be easily held or remembered.

It is apparent that, in general, the more elementary the idea the harder it is to define, since it is harder to find still simpler ideas in terms of which to express its properties. This is very true in arithmetic, for here the ideas of unit, number, etc., are so fundamental that it is almost impossible to find definitions that serve either of the two purposes for which they are made.

In all probability no child ever gets the idea of number from a definition of it. Pupils for the most part come to school with

ideas of the integers at least, and the number concept enlarges with every new type of number encountered. In view of these facts definitions should be used very sparingly in elementary arithmetic. In fact they should be used only where they serve at least one of their two purposes.

If one takes up some of the arithmetics in common use he will find that an attempt is made to define most of the fundamental ideas. And such attempts as some are! The very subject which above others should lead to habits of accurate thinking seems to be the occasion for a display of the very opposite on the part of many writers on elementary mathematics, and particularly on arithmetic.

The definition that "Arithmetic is the *science* of numbers and the *art of* their computation" which is sometimes given would hardly satisfy either of the two purposes of a definition given above. The terms science and art are not comprehensible to the student of elementary arithmetic.

Number is an idea and is essentially abstract, a fact too often lost sight of. Some numbers have physical realities corresponding to them, but the numbers themselves are abstract. In view of this, and of the fact that the converse of a definition must be true the nonsense of the definitions of unit and number often given, even in some of our most recent arithmetics, is apparent. To say that "unit is a single thing" and that "A number is a unit or collection of units" comes very near the limit of absurdity even though we add to the end of the latter "of the same kind."

Another definition, which seems to be a relic of earlier stages of civilization when the race knew integers only, is that of multiplication as "a short process of repeated addition" or perhaps the more recent but practically equivalent one that "Multiplication is the process of taking one number as many times as there are units in another" which does not work very well when the multiplier is a fraction. Other examples might be given but these few will suffice to show that much defining is out of place in elementary arithmetic and such types are out of place anywhere.

A MNEMONIC FOR CERTAIN TRIGONOMETRIC IDENTITIES.

BY L. H. RICE.

While looking over identities recently, I was struck with the advantage of using a certain set of equations, and therefore with the need for some simple way of memorizing them. These equations are:

$$\sin x = \cos x \tan x = \cos x/\cot x = \tan x/\sec x,$$
$$\cos x = \cot x \sin x = \cot x/\csc x = \sin x/\tan x,$$
$$\tan x = \sin x \sec x = \sin x/\cos x = \sec x/\csc x,$$
$$\cot x = \csc x \cos x = \csc x/\sec x = \cos x/\sin x,$$
$$\sec x = \tan x \csc x = \tan x/\sin x = \csc x/\cot x,$$
$$\csc x = \sec x \cot x = \sec x/\tan x = \cot x/\cos x.$$

Some arrangement of the names of the functions, it seemed, might be devised which would afford a mnemonic; and the functions did, in the end, prove tractable, giving the following diagram.

$$\begin{array}{ccc} & \tan x & \\ \sin x & & \sec x \\ & & \\ \cos x & & \csc x \\ & \cot x & \end{array}$$

Take any function in this circle of functions and read the two adjacent functions as a product equal to that function. Or, read one of these adjacent functions *over* the next one beyond it as a quotient equal to the selected function. Or, read the other of the adjacent functions *over* the next one beyond that as another quotient equal to the selected function.

In proving identities, the reverse process comes often into play. The product of two functions being seen to consist of two functions that are separated by one function in the circle, that one is substituted for the product. Or, a quotient being seen to consist of two functions that are successive functions in the

circle, the equivalent single function on the proper side of them is substituted.

Beginners in trigonometry have no trouble in learning the circle and its use, and mentally apply it when effecting transformations, and the work thus done has a heightened interest for them and for the class.

SYRACUSE UNIVERSITY.

THE REQUISITE.

In education, various books and implements are not the great requisites, but a high order of teachers. In truth, a few books do better than many. The object of education is not so much to give a certain amount of knowledge, as to awaken the faculties, and give the pupil the use of his own mind; and one book, taught by a man who knows how to accomplish these ends, is worth more than libraries as usually read. It is not necessary that much should be taught in youth, but that a little should be taught philosophically, profoundly, livingly.—*William Ellery Channing.*

THE WORK OF AN ENGLISH MATHEMATICAL STUDENT.

By H. Bateman.

A few weeks ago Professor Babb invited me to give you a short address on some topic connected with the study of mathematics at English schools and universities. In thinking over the subject I have naturally tried to recall the various impressions I received during my school and college days, leaving out, of course, the marks of the cane. For a little while it has seemed just as if time had been put back ten years or so, that I was again looking at things from the point of view of the student and comparing experiences and opinions with my college friends.

My school experience has not been sufficient to enable me to give you a representative account of the methods of teaching adopted at the different public schools and of the amount of preparation which the boys received, because circumstances made it necessary for me to attend one school at a time.

What little information I have to give you, then, apart from my own observations, has been derived from conversations with friends or from more reliable sources, such as periodicals and addresses given by prominent men.

At a school like St. Paul's, London, or Manchester Grammar School, mathematics is studied both as a routine subject and as a specialty. The student who specializes in mathematics does so principally with the object of gaining a scholarship at one of the universities; he must consequently acquire considerable facility in solving problems and in being able to reproduce bookwork quickly and accurately.

The plan which is adopted at some schools is to leave the student practically to himself except for occasional help from the master in the solution of a difficult problem. Very few lectures of an advanced character are given, partly because the master's time is occupied with elementary work and partly because the few students who are specializing in mathematics are at different levels, both as regards knowledge and ability. At

other schools regular courses may be given; for instance, I was told of one school where a master gave an excellent course on dynamics, illustrating the theory by means of simple experiments.

Generally when a boy gains a scholarship at Oxford or Cambridge he has a fair knowledge of synthetic and analytical geometry, higher algebra, including the theory of equations, higher trigonometry, elementary differential calculus and mechanics.

Of course, I don't want you to infer from this that the different students who go up to Oxford and Cambridge have all received about the same amount of preparation. The mathematical equipment of a Cambridge freshman is a quantity which varies over a very wide range. Under the old regime the tuition a student received would depend entirely upon the amount of preparation he had received and the impression he had given of his ability. If he was well equipped he was advised to attend lectures of an advanced nature straightaway; these might consist of analytical statics and differential equations in his first term, with elliptic functions and particle dynamics in his second term. He thus spent his first two years preparing for what used to be the second four days of the tripos examination.

At the smaller colleges it is different, and generally a man will spend his first year preparing for the first four days of the examination. This plan is probably still followed now that the two parts of the examination are taken in different years and are slightly different in character from what they were previously.

A boy who is trying for a scholarship at one of the modern English universities like Manchester, Liverpool or Birmingham will not require such an extensive knowledge of mathematics, and will generally leave school a little earlier than if he were going to Oxford or Cambridge. In the scholarship examination the subjects are geometry, the substance of Euclid I.–IV. and VI.; algebra as far as the binomial theorem inclusive; plane trigonometry to the solution of triangles; elementary analytical geometry; the conic sections. Credit is given to candidates for their knowledge of classics: elements of English language, literature and history, French and German.

In his first year at the university the student attends courses on subjects not belonging to his specialty, so as to get a good general education, but later on he specializes entirely.

At Cambridge the plan is different. An honor man who is reading for the tripos is recommended to devote all his time to his special subject; in fact, the mathematical student has so much work to get through that it is almost necessary for him to coach both in term time and during the part of the long vacation which he spends at college.

I might say a few words here about the general effect of the coaching system.

The students used to vie with one another to see who could produce the prettiest solution of a given problem. The geometrical questions usually provided most scope for originality and on one occasion a student showed up a geometrical solution of a difficult analytical problem which was obviously meant to be done analytically. The coach gazed at it and said: "Oh! I am sure I couldn't have done that."

One influence which the Cambridge training has upon mathematical students is to develop the artistic faculty; a good man will not be satisfied with any old solution of a problem and when he has got one solution he will try to simplify it and present it in as clear a form as possible.

It used to be very amusing to see the look of pain that would come on the coach's face when a man showed up a six-page solution of a problem which could be solved very easily in two pages. Mind you, credit was always given to a student who obtained a solution of a difficult problem, whether the solution was pleasing or not, but brevity received due encouragement.

E. J. Routh, the man who beat Clerk Maxwell in the tripos examination, is perhaps the best known of the Cambridge coaches, and his method of teaching may be regarded as typical. The following description of Routh's method of teaching is taken from Forsyth's obituary notice of him in the *Proceedings of the London Mathematical Society*.*

The pupils went to him three times a week, for an hour on each occasion. They were arranged in classes of six to ten, selected at first mainly by their own statements of mathematical knowledge brought from school, and sifted later by their attainments and progress. Each hour's work was substantially of the nature of a lecture. A few minutes at the beginning of the

* Ser. 2, Vol. 5 (1907).

hour were devoted to the inspection of written exercises, a process at which Routh was wonderfully quick. The remainder of the hour was occupied with a skilful exposition of the subject under consideration, sometimes summarized in such a way as almost to compel remembrance.

In addition to the direct teaching, he sent to all his pupils a weekly paper of a dozen problems, taken mainly from past examinations. The conditions of answering this paper were made to alternate in successive weeks. In one week a student could devote unlimited time to the questions; in the other he was expected to limit himself to three hours so as to feign some similarity to the pressure of the Senate House. The pupils' answers were deposited in Routh's rooms in Peterhouse on the Friday or the Saturday of any week; they were returned on the succeeding Monday, when he placed in an outer room a carefully written set of solutions.

To add a stimulus of competition, every pupil's answers had been marked, and there was a list containing names and marks, so that the men even of different years were almost encouraged to regard the weekly problem paper as a field for rivalry and comparison.

Finally, at some period before the crucial examination, there began the process of revision, in the form of bookwork papers. The answers to each such paper had to be brought two days after the paper was set, and they were examined critically in class. The pupils were brought up to the mark, so far as the teacher was concerned, partly by good humor, partly by occasional warnings that were kindly in phrase and in spirit, and always under a genial sense of his easy mastery over the whole range of subjects included in the examination.

It is obvious that this process, if not supplemented by other means, would imply a uniformity of treatment which would not be found advantageous by the eager members of even an average class. The necessary supplement was provided by Routh in the form of "manuscripts," which really were of the character of small pamphlets dealing with special portions of subjects not taught in classes. Pupils were expected to copy these manuscripts, in default of English text-books; but diligence in taking copies was far from universal.

The coaching system is probably despised by some of the continental mathematicians and I remember an amusing story about a foreigner who came to lecture in Cambridge on chemistry. He wanted to impress upon the student that he was going to interest them in research work and not to coach them up for an examination, so he said: "I want you to understand that I am not a coachman."

The subjects for the first part of the mathematical tripos under the old regulations were:

Pure geometry, including geometrical conics.

Newton's Principia.

Algebra and trigonometry.

Analytical geometry, including some of the elementary properties of higher plane curves. Solid geometry, including elementary differential geometry.

Differential and integral calculus.

Elementary statics and dynamics.

Analytical statics and attractions.

Particle and rigid dynamics.

Optics and astronomy.

Electricity and magnetism.

Hydrostatics and heat.

Hydrodynamics and sound.

Differential equations, elliptic functions, Fourier's theorem, spherical harmonics, Bessel's functions, calculus of variations.

Under the new regulations an examination in the elementary subjects can be taken at the end of the second term. This examination is now called Part I. Part II. comprises subjects in schedules *A* and *B*. Schedule *A* contains the ordinary subjects to be taken by all candidates, the *B* subjects being taken only by men who are candidates for a mark of distinction. Part II. cannot be taken earlier than in the eighth term;* it corresponds more or less to the second four days of the old examination, but a candidate has the option of specifying the range of subjects *B* in which he desires to be examined. The order of merit has now been abolished and the names of the successful candidates for honors are arranged in three classes: wranglers, senior optimes and junior optimes, the names in each being in alphabetical

* See *Nature*, January 17, 1907, or the Cambridge Univ. Calendar.

order. A distinctive mark is attached to the name of a man who has done fairly well in schedule *B*, and a different mark if he deserves special credit for his answers to *B*.

Under the old system a man generally spent three years in preparing for his tripos, but occasionally a man who had graduated at some other university would take the examination at the end of two years. I can remember how astonished we all were when Eddington did this and came out at the head of the list; after this feat had been performed once it was repeated more than once.

The old Part II. of the mathematical tripos has now been abolished and students are left to study the higher branches of mathematics without being bothered with preparation for an examination.

A student at a university like Manchester and Liverpool will not cover quite so much ground as one at Cambridge, but he will nevertheless receive a good training in both pure and applied mathematics; astronomy is generally omitted from the list of subjects. In some cases a course of lectures will be given on some subject like the theory of functions or the theory of probability, which will give the students a glimpse of some of the developments which are taking place in the mathematical world.

I am not very well acquainted with the teaching of mathematics at Oxford University. As far as I know the examination system is very similar to the one now adopted at Cambridge; there is an examination called Moderation, which is taken in the first or second year, and a final examination, which is taken in the third year. As regards the work done in pure mathematics, considerable attention seems to be paid to geometry and the algebra of quantics. The principal laurels to be won by a mathematical student at Oxford are the junior mathematical scholarship, the senior mathematical scholarship and a fellowship.

With regard to the present state of school teaching* in mathematics, I know very little. You will remember that about ten years ago a movement in the direction of reform in mathematical

* Some useful information on the teaching of mathematics in the United Kingdom may be derived from the eight reports recently issued by the Board of Education and published by Wyman. The reports issued by the Mathematical Association should also be consulted.

teaching was set on foot by Professor Perry and others. The abolition of Euclid as the sole authority in geometry and the stress laid upon graphical methods have been direct consequences of the agitation, and the changes in the mathematical tripos at Cambridge are also partly due to it. The aim of the movement* has been the very practical one of making the mathematical teaching more suitable for the training of engineers and students of natural science, and also to relieve the English secondary school teachers from the burden of a too precise examination system, imposed by the great examining bodies.

It will be interesting to make a survey in a few years' time to see how the character of English mathematical work has been affected by these changes.

The attitude of some of the teachers towards the new methods is indicated in some of the discussions and addresses given at meetings of the Mathematical Association,† and also in a discussion which took place at a meeting of the Manchester Mathematical Society, in 1908. At this meeting Professor Leahy read a paper on the desirability of a definite order of propositions in geometry. He thought that a definite order differing very little from those adopted in the new text-books could easily be decided upon and would be welcomed by most teachers and students. He was supported by the majority of the members present; but some thought that so many improvements had been made in recent years that if a definite order were introduced at the present time progress would be retarded.

BRYN MAWR, PA.

* A good account of this movement with references is given in Prof. E. H. Moore's presidential address to the American Mathematical Society. *Bull. Amer. Math. Soc.*, Vol. 9, 1902–3.

† *Mathematical Gazette*, Jan., March, Dec., 1909, March, July, 1910, March, 1911.

PRELIMINARY REPORT OF THE ARITHMETIC COMMITTEE.

To the Association of Teachers of Mathematics in the Middle States and Maryland:

When the arithmetic committee was appointed, its work was outlined as follows:

"(1) To give a careful and detailed statement of the objects and aims in teaching arithmetic. These objects should be expressed in specific and not in general terms. (2) To consider how these objects are best attained. Experiments of value so far as possible should be employed."

Your committee during the past five months has been occupied in considering the first of these instructions, namely, the objects and aims in teaching arithmetic. So far as the introductory work is concerned, the object or aim is clear, therefore arithmetic in grades I. to IV., according to the common arrangement, has been eliminated from our study. Difference of opinion, however, arises as soon as the foundation work is completed. Therefore the committee has confined its inquiry to the consideration of the prime reason for teaching arithmetic after the elementary operations with integers and fractions have been mastered.

A questionnaire was prepared and sent to fifty-five persons, including experts in the fields of psychology, pedagogy, and mathematics. Twenty-five replies were received which have been summarized as follows:

Synopsis of Replies to Question I.

I. What is the prime reason for teaching arithmetic in grades V. to VIII. as our schools are generally organized?

To master and give exactness and speed to the fundamental operations taught below the 5th grade—and to show how these operations may be applied to problems of daily life, both of civic and local interest.

To acquaint them with the quantitative side of large industries and other phases of social life, such as insurance, banking, etc.

Robt. J. Aley, President, University of Maine.
A. B. Poland, Superintendent, Newark, N. J.
E. L. Thorndike, Teachers College, New York City.
W. W. Hart, University of Wisconsin, Madison, Wis.
D. E. Smith, Teachers College.
F. M. McMurray, Teachers College.
A. H. Huntington, Central High School, St. Louis, Mo.
H. E. Slaught, University of Chicago.

To strengthen one's understanding and appreciation of the activities of the business world by the study of denominate numbers, interest, partnership, taxation, stocks and bonds, and similar topics, as it is equally important for the pupil to have an understanding of the business environment as of the natural environment—practically essential in the equipment for life work of every individual.

Julius Sachs, Teachers College, New York City.
F. E. Downes, Superintendent, Harrisburg, Pa.
W. C. Bagley, University of Illinois, Urbana, Ill.
D. E. Smith, Teachers College.
David Felmley, Normal, Ill.
Joseph V. Collins, State Normal School, Stevens Point, Wis.

For those who go to high school and college, algebra and geometry should begin with the seventh grade.

For pupils not going to the high school, there should be a course (in the 7th and 8th grades) in elementary economics and sociology,—buying and selling of goods, insurance, taxes, partnership, etc.

Henry Suzzallo, Teachers College.

To render conscious all those mental factors and activities, control of which gives an individual the power to attack with confidence and with more than average success any problem in any field that it may be necessary for him to attack; in other words, which give him general ability.

There is no question of transfer of specific training here. Experiments have proved that specific training transfers little

if at all, even in the same field. But specific training may generate ideas, or ideals whose influence will transfer to every field. Mathematics should make conscious those ideals whose influence is most far reaching in their effects upon reasoning in any field.

S. A. Courtis, Detroit, Mich.

To learn when to add, subtract, multiply, and divide in problematic situations calling for two or more of these processes; to learn the rudiments of mathematical thinking; to acquire a regard for, and the habit of neatness, orderliness and precision; and to learn a little of the connectedness and unity of arithmetical subjects,—not the theoretical ones, not the non-essential ones, not the complex ones.

G. W. Myers, University of Chicago, Chicago.
E. P. Sisson, Colgate Academy, Hamilton, N. Y.

To develop and strengthen the reasoning powers of our girls and boys and thus give them the ability to reach safer and sounder conclusions in their affairs in life,—the disciplinary value.

E. P. Sisson, Hamilton, N. Y.
Albert H. Huntington, Central High School, St. Louis, Mo.
W. H. Metzler, Syracuse University, Syracuse, N. Y.

In order that people may measure mass, time, energy, space, and value; to adjust means to ends in the practical affairs of life.

David Felmley, Normal, Ill.

To insure, through proper drill, a complete and permanent mastery of simple operations with integers, common fractions, and simple decimals; also simple operations with percentage and simple tables of measurement, together with the beginning concepts of algebra and the elementary geometrical relations.

Edward O. Elliot, University of Wisconsin, Madison.
F. E. Downes, Superintendent, Harrisburg, Pa.

To cultivate general mental alertness.

H. E. Slaught, University of Chicago.

Synopsis of Replies to Question II.

II. What is the greatest need for reform, if we are to accomplish this result?

(*a*) Wise selection of material.

(*b*) Elimination of all unpractical problems.

(*c*) Raising the standard of attainment of pupils.

Joseph V. Collins, State Normal School, Stevens Point, Wis.

There is need for a reduction in the topics suggested by the text-books and a general simplification of method. The teachers above all ought to acquire a mastery of every subject which will make them independent of the text-book. Applied arithmetic, that is to say, such arithmetic as actually and generally occurs in the lives of our pupils, ought to be the standard for the work set. Unduly complex problems are useless and absurd. It would then be possible to introduce many practical questions in mensuration, a combination of drawing and simple calculation of space and volume contents could be combined with the arithmetic.

Julius Sachs, Teachers College, New York City.

To make a sharp differentiation at the close of the sixth year, aiming to complete the formal fundamental work within the first six years, and to make the purpose, content and method of instruction of the seventh and eighth years vastly different.

Henry Suzzallo, Teachers College, New York City.

The elimination of the useless and antiquated material from the arithmetic is the first step necessary to a better program. The introduction of present day material and the adjustment of this material to present business practice is the second step. The third step is the better preparation of teachers by the normal schools.

R. J. Aley, President, University of Maine.

Ellwood P. Cubberley, Leland Stanford University, California.

The greatest need for reform is to close the elementary work at the end of the 6th grade and begin the high school work with grade seven.

David Eugene Smith, Teachers College, New York City.

Make the teaching as practical as possible, insist on clear analysis, full comprehension. The work should not drop into mere mechanical routine or memory exercise.

J. G. Estill, Hotchkiss School, Lakeville, Conn.

A. B. Poland, Superintendent, Newark, N. J.

More arithmetic of the mental type.

W. H. Metzler, Syracuse University.

H. E. Slaught, University of Chicago.

Simplification and humanizing of the example material, making it more nearly the sort met in actual daily life.

Walter W. Hart, University of Wisconsin.

H. E. Slaught, University of Chicago.

The specific answer to this question is that the course of study should be reorganized on the basis of the fundamental aim. Efficient teaching of fundamental abilities needed for social service must come before everything else. Simplification of the course of study, quantitative measurement of the powers and capabilities of the growing child, and of the effects of teaching effort, together with experimental determination of efficient methods, is my program. The higher aims will become clear as the lower training becomes more specific and efficient.

S. A. Courtis, Detroit, Mich.

Scarcity of teachers who know enough arithmetic and of real scientific method to teach the subject even fairly well; and scarcity of scientific texts together with the general failure to choose texts in arithmetic with an eye single to educational interests.

G. W. Myers, University of Chicago.

The main means by which this can be accomplished is to make a study of such topics on the quantitative side and teach the facts through series of problems; each of whose answers is worth finding out and which together tell a story.

F. M. McMurray, Teachers College, New York City.

Teachers that have a broader and deeper knowledge of arithmetical number and its application to the problems of life.

Teachers who will teach the subject rather than a book, and who use the book as a means to an end. I would have arithmetic taught but three years in our grades and one year in the high school.

E. P. Sisson, Hamilton, New York.

The greatest needs seem to me to be a scientific study of just what is now accomplished in the last four grades, a study of the uses to which arithmetic is actually put by the pupils who attend these four grades and the organization of subject matter and methods of teaching unhampered by tradition.

Edward L. Thorndike, Teachers College, New York City.

Lead teachers to see that what we may call the computation element of the so-called arithmetic of the upper grades is secondary, that the primary purpose of the teacher is to acquaint the children with a great variety of human affairs to which arithmetic is applied. Many of these affairs will be somewhat remote from the life of many of the pupils, but there is a great body of common knowledge that everybody needs connected with areas and volumes, specific gravity and business life that all should be taught.

David Felmley, Normal, Illinois.

Eliminate all subjects of doubtful or little "knowledge" value, especially those having little "incidental" value,—such as many odd and unnecessary rules of measurements involving and requiring memory only, Duties and Customs, Annual Interest, Partial Payments, Stocks and Bonds, Domestic and Foreign Exchange. Teach those subjects with greatest intensity that bear more directly upon the incidental results to be accomplished. Bear especially upon subjects that may be regarded as preliminary to the study of algebra and geometry, such as Involution, Evolution, Ratio and Proportion, and teach them so far as feasible with a view to paving the way to this more advanced work. Bear hard upon subjects most common in special business relations. Make problems practical and related to the industrial and commercial life of the community.

F. E. Downes, Superintendent, Harrisburg, Pa.

Reform ought to bring about exactness and habitually accurate thinking.

A. H. Huntington, Central High School, St. Louis, Mo.

The replies are suggestive of the trend of thought on arithmetic teaching in the upper grades. The committee feels that it is not at present justified in making final recommendations to the Association. The reports of the International Commissioners on the Teaching of Mathematics will soon be completed. These reports, together with other comparative studies with which the committee is in touch, should be consulted before any final recommendations are proposed.

The committee therefore suggests that this preliminary report, including the summarized replies to the recent questionnaire, be

published in the MATHEMATICS TEACHER for the information of the Association.

Respectfully submitted,

JONATHAN T. RORER, William Penn High School, Philadelphia, Pa., chairman,

M. J. BABB, University of Pennsylvania, Philadelphia, Pa.,

J. C. BROWN, Horace Mann School, Teachers College, New York City,

A. M. CURTIS, Normal School, Oneonta, N. Y.,

CHARLES C. GROVE, Columbia University, New York City,

EMMA WOLFENDEN, William Penn High School, Philadelphia, Pa.,

Committee.

PHILADELPHIA, November 30, 1912.

CERTAIN PROBLEMS IN THE TEACHING OF SECONDARY MATHEMATICS.*

By David Eugene Smith.

It is not without considerable hesitancy that I come before an association like yours to speak upon some of the great problems that confront us in the teaching of secondary mathematics. This hesitancy arises from several causes, prominent among them being the feeling that I shall only be "carrying coals to Newcastle." For surely these problems are already in your minds, and many of you have pondered over their significance and their solution quite as seriously as I have and no doubt with a more satisfactory issue. I hesitate, also, because I can merely state them with no attempt at solution, mindful all the time of the ancient adage referring to questions which a wise man can not answer. But after all, there is a value in clearly stating from time to time the large questions that confront our guild, for if problems were never formulated they would never be solved, and it is upon associations like this that we must largely depend for the solution of the ones that I shall venture to lay before you.

I. Has the Day of Secondary Mathematics Passed?

The first of the great questions that confront us at the present time relates to the very existence of secondary mathematics in our curriculum. To many of us it may seem preposterous that the question should seriously be asked. We say to ourselves that if anything is to be blotted out let it be some language, or let it be any one of the various manual arts that are from time to time exploited only to find, in most cases, an early resting place in the great educational necropolis of forgotten graves.

But the question can not be dismissed in any such way as this. Other subjects have been seemingly as well entrenched as mathematics, and yet they have passed away. Formal logic was at

* An address given before the New England Association.

one time one of the great features of a liberal education; it gave place, in the secondary school, to formal grammar; but a university course in formal logic is now a rarity, and a high school course in formal grammar, as it was conceived of a few years ago, is almost unknown. The world seems to proceed as well without them as it did when they held prominent place, and we have to face the question whether it would not get along just as well if algebra and geometry followed them into educational oblivion. The medieval Compotus was once an essential feature in the education of a learned man; it was apparently entrenched in a position of security; and yet, as I mention it to-day, half of this audience may be ignorant, and excusably so, of even the meaning of the word. Somebody at some time asked the question, "Why should an educated man need to study the Compotus?"—and the answer came in due time, and the subject was soon forgotten. Somebody to-day raises the question, "Why should an educated man need to study algebra?"—and we, the teachers of mathematics, must answer. A high school in which I am interested holds its last class in Greek this year,—one of the best-known high schools in the country, a school of five hundred selected pupils, all of whom are hoping to enter college. Shall it be that, a few years hence, this same school shall be teaching its last class in geometry, compelled to drop the subject because it is no longer demanded?

We may say to ourselves that high school mathematics has always existed, that it is not a college subject, and that it is absurd to talk of abolishing it. But if we say this we forget that the American high school is itself a new institution; that it has no exact parallel in other countries; that other countries select their pupils for secondary work while we seek to educate the mass; that 95 per cent of our high school pupils do not go to college, and do not hold the intellectual standards held by the boys of our old academies; and that it is comparatively a recent demand of the American colleges that its candidates must offer any mathematics beyond arithmetic. So when we speak of high school mathematics we should bear in mind that our high school has not as yet proved its worth, and is even now being weighed in the balance with rather unsatisfactory results, and that pre-collegiate mathematics is only a recent matter. If

a pupil postpones his Greek until he enters college to-day, why should he not postpone his Latin, his algebra, and his geometry to-morrow?

Now this is not a cry of alarm for the sake of temporary effect; it is a succinct statement of the arguments that we frequently hear to-day from the general educator. Up and down this country, before hundreds of gatherings of teachers, the question is being vociferously propounded, "Why should the girl ever study algebra?" Even in associations of teachers of mathematics the question is being asked, "Who would stand to-day for the spirit of Euclidean geometry?" And the men who ask these questions hold prominent positions; they are professors in universities, educators of influence, men whom the mass of teachers naturally look to as leaders. The problem is, therefore, a real one and one that we have to face.

But we must not deceive ourselves by thinking that we can successfully meet it by mere opprobrium. We not infrequently hear it asserted that the general educator is usually a man of a low degree of scholarship, that his range of culture is very limited, that he was taught Latin so poorly that he believes it can never be taught in any other way, and that he rarely stands for any intellectual ideals; but we must remember that, even if the assertion has some truth, enough people of this type might easily create a Zeitgeist that would not down by any such formula as "Weave a circle round him thrice." These men who attack the ancient culture have been rather recklessly called educational muck-rakers, men who seek only the bad and judge everything by that; they have been rather hysterically denominated pedagogical anarchists, men who destroy without rebuilding; and they have been frequently looked upon as intellectual iconoclasts, those who in their zeal to destroy idols are willing that all the beauties of art, which their dull vision fails to see, should go to the scrap heap. In all these assertions there may be some grain of truth, but no one gains anything by giving expression to such an attitude of mind. Such expression merely brings the countercharge that those who hold other views are reactionaries, *laudatores temporis acti,* unprogressive, and selfish clingers to their little jobs. The epithet of "old fogy" is as weighty as that of "muck-raker."

When we calmly consider the question, we find that it relates to the value of algebra and geometry for the democracy that America, in distinction from the rest of the world, is trying to educate in the high school. What does democracy want of mathematics? And in our America of the dollar we find that the question is often reduced to that of the immediate utility of algebra and geometry. The potential utility does not seem to enter into the consideration of the type of reformer that seems to have the most to say upon the subject. And here appears to be the real point at issue: one side demands the immediately useful, while the other stands for that which it claims to be potentially so. Can we, therefore, justify our secondary mathematics on the potential side? For surely no one would for a moment claim that the teaching of the immediately practical part of algebra would require more than a month, or that the immediately practical part of demonstrative geometry exists, taking these words in their usual popular significance. Such, then, is the first of the large problems that seem to loom up before us. Call it a claim for mental discipline if you please,—this is a mere question of fashionable or unfashionable phraseology; it is a claim for serious attention to a vital issue in education.

II. Why Should Not All Mathematics be Elective?

The general educator is usually found to concede that mathematics should be taught in our high schools, but he has frequently been heard to assert that it should be elective. Many teachers of mathematics, perhaps most of them, would personally welcome such a change, since the pleasure of teaching is largely increased if the learner takes the subject *con amore*. But of late a new type of educator has appeared, the one who proposes to weigh in psychological scale the intellect of youth and to guide him aright. You have it in your own part of the country to-day in the phase of "vocational guidance," and in this work some excellent people are seriously engaged, so many, indeed, that we are likely to see it appear generally. Let the boy who gives promise in science begin his specialization early, and the one who takes to Latin bend his energies there. Let the one who, by virtue of his surroundings and family, is destined

to be a hewer of wood, early come to like to hew, and let him be taught the nobility of labor with the hand.

Let me tell you some advice that I have given within a few years past in cases like these, and ask you if I did not act with becoming pedagogical wisdom.

Not long ago there came to me a father who wished to train his boy for trade in a seaport town, and who asked my advice as to the proper education to give him. The problem seemed simple. The community was not an educated one; it lived off its little shipping industry; the boy was destined to small business and to small reward; he gave no promise of anything better, and the advice was, therefore, unhesitatingly given that the only mathematics he needed was arithmetic through the sixth grade.

Another parent asked me a little later about his son. The boy was of the ordinary type and would probably follow his father's occupation, that of a sculptor. What mathematics would it be well for him to take? I suggested a little study of curves, some geometric drawing, and the modeling of the common solids,—a bit of vocational guidance that seemed to me particularly happy.

A third boy happened to be with me on a steamer and I took some interest in talking with him and with his mother. They lived in a city of no particular note, at any rate at that time, and the boy was going into the selling of oil within a few years. The profits of the Standard Oil Company appealed to the family, and I advised him to learn his arithmetic well and get into business as soon as he could.

Out of the store of my memory I recall a curious lad whom I came to know through my sympathy with the family. The mother was a poor woman and she took the boy, when little more than a baby, over to Riverside Park one day when there was a naval parade. A drunken sailor, having had a fight with a group of hoodlums, rushed through the crowd of spectators and slashed right and left with a knife. In the excitement the boy, in his mother's arms, was horribly cut in the face. When I knew them he was about ten years old, unable to speak plainly, and already a misanthrope through his affliction. I advised the mother to give the boy a vocational education, telling her that through the use of the hands he would satisfy his desire for

motor activity, and that this would compensate him for the loss of verbal fluency and would tend to make him more contented with his lot, and in this advice I feel that I would have the approval of educational circles.

And, finally, out of this series of experiences, let me recall the case of a boy whom I came to know through a noble priest who found him one morning, an infant of a few days of age, on the steps of his church. We talked over the best thing to do for such a foundling, one who, at the time I knew him, was in the primary grades. He showed no great promise, he was without family recognition, and his only chance, apparently, was in the humbler walks of life. I recommended a vocational school where he could quickly prepare for the shop or the lower positions of trade, and the good priest approved my plan, although he finally followed one that was quite different.

Of course, it is apparent that I have here spoken in parables. Perhaps you already recognize the boys, and perhaps you feel how sadly I blundered in my counsel. For the first of these whose cases I have set before you felt a surging of the soul a little later, and this was recognized in time, and he became one of the Seven Wise Men of Greece, Thales the philosopher, he who introduced the scientific study of geometry into Greece. The second felt a similar struggle of the soul and his parents recognized my poor counsel in time to save him and to give to the world the founder of its first university, Pythagoras of Samos. The third boy, for whom only the path of commerce seemed open, and this in a town only just beginning to be known, was the man who finally set the first college entrance examination, the one who wrote over the portal of the grove of Academos the words, "Let no one ignorant of geometry enter here,"—Plato, the greatest thinker of all antiquity. The fourth, the hopeless son of poverty, maimed, sickly, with no chance beyond that of laboring in the shop for such wage as might by good fortune fall to his lot, became the greatest mathematician of his day,—always the stammerer (Tartaglia), but one whom Italy has delighted to honor for more than three centuries. And the last one of the list, the poor foundling on the steps of St. Jean-le-Rond in Paris, became D'Alembert, one of the greatest mathematicians that France, a mother of mathematicians, ever produced.

Shall we advocate the selection of those who are to study mathematics and close the door to all the rest? Are we so wise that we can foresee the one who is to like the subject, or succeed in it? Have we so adjusted the scales of psychology that we can weigh the creases in the brain, or is there yet invented an X-ray that will reveal to us the fashioning of the cells that make up its convolutions?

Of course, it will at once be said that these illustrations that I have given are interesting, but that they are unfairly selected; that those boys gave earlier promise in mathematics than I have said. It will be asserted that I should have taken the case of the stupid boy, the one who did not like school, the one who liked to play with little wind wheels, who liked to fight, who actually did run away from school, and who stood near the bottom of the class in mathematics. Such a case would be a fair one, one in which we could safely say that prescribed algebra and geometry is out of place. And I suppose we must agree to this and confess that the argument from the historical incidents that I have mentioned was unsound. Let us rather take the case that I have described, and let us see to whom the description applies. I need hardly tell you who this boy is; he is well known to you; he is well known to the world; and long after every educational reformer has passed into oblivion his name will stand forth as one of England's greatest treasures, for it is the name of Sir Isaac Newton.

But, again, I have been unfair, perhaps. I should have taken positively hopeless cases, for such can surely be found. I should have taken some illiterate man, one who does not learn to read until he is nearly out of his teens, or else some man who shows no promise in mathematics by the time he reaches manhood, or someone who by the time he is thirty is to show no aptitude in the science. It is so easy to theorize! But let us have care, for the men whom I have now described are Eisenstein, Boole, and Fermat. Take them away and where is your theory of invariants, your modern logic of mathematics, and the greatest genius in theory of numbers that the world has seen?

But I am wandering afield, and I fear I may be interpreted to question the modernizing of our educational work. I thoroughly favor industrial education when it is not so narrow as to con-

demn a boy to some particular groove in life, and I earnestly hope that we shall so guide our youth that every boy and girl will leave school fitted to do something well. I do not believe that any thoughtful educator wishes to guide a pupil in a narrow path nor keep from him the chance that the world owes him. It seems right, however, to set the problem clearly before us,—Can we safely say that we may close the door of mathematics to any boy? Should he not be given the chance? If he fails, that ends it, but if he succeeds the world is the winner in the lottery. Of course, this does not answer the question as to what this chance should be; it is quite possible that it should not be our present algebra; it is even possible that it should be merely some form of mensuration that masks under the time-honored name of geometry; it may conceivably be some emasculated form of fused mathematics that has none of the logic of geometry and none of the beauty of algebra, although I do not believe it; and it may even be some form of technical shop mathematics that appeals to but few pupils because of its very technicalities. This is the part of the problem to be solved. But that the door of mathematics of some substantial character shall not be opened, and opened after arithmetic has been laid aside as the leading topic, seems unthinkable.

III. Does the Girl Need to Study Algebra?

A third question that seems at the present time to agitate the educational interests in some parts of the country relates to the study of algebra by the girl. This carries with it the corollary that no mathematics whatever, beyond mere computation, is to be required except, perhaps, of the boy.

How this meets the views of the emancipated woman I do not know. I assume that she would say, if asked, that if algebra and geometry are good for the boy, save in the narrowest technical sense, they are good for the girl also. I should think she would say that if mathematics is the one subject that makes us understand our infinitesimal nature in the infinite about us, if it is the one science that has had the most to do with banishing the superstition that comes from simply looking at the heavens with lack-lustre eye, if it brings a mental uplift that no other science brings and lets us see what seems the nearest to exact

truth of anything that we meet, and if we find at every turn the mathematical invariant, a timely symbol of the unchangeable presenting itself as the youth passes into manhood,—I should think that she would say that if mathematics brought these things in its train it is worth while for the girl if it is for the boy.

But to me there is a more serious side to the problem. Mathematics is going to continue to be taught in the schools. It will gradually change to meet the demands of the times as it has always done. So far as we can see there will be a mathematics that is immediately practical for those who are not hoping for any intellectual leadership, and there will be the mathematics that I have described as potentially useful for those who are not content with remaining in the lower intellectual class. But in any case there will be mathematics, and the boy will study it. The question as to the girl is, as I have intimated, a more far-reaching one. When this was an agricultural country the father directed the education of the children. In the long winter evenings he had the time and inclination to help those of his household who at that season of the year were taking what was termed their schooling. The mother had other duties that filled her time, and, moreover, was not herself well enough educated to give much assistance in the matter of study. But America has changed. With our urban population at least, the father no longer has control of the education of the children. In our manufacturing centers he is busy in the shop, and his hours are no longer limited by the light of day. On the other hand, the urban mother no longer weaves and spins, no longer helps in the fields, no longer preserves fruit, and has almost forgotten how to make bread. She has more time for the higher life, and it is she, rather than the husband, who gives the direction, the help, and the encouragement in the education of the son as well as the daughter. The father may need some algebra in his trade, for if he reads the artisans' journals he must know how to manipulate formulas, but the mother must have a general knowledge of the subjects that children study if she is sympathetically to direct their intellectual activities. It is the woman quite as much as the man who needs the broad education at the present time.

Hence the problem seems to me not so much to decide whether

or not the girl should study algebra, as to decide how we shall so teach the subject to her that she will know of its beauties, of its purposes, and of the feeling of mastery that comes from its pursuit. Such a problem may well occupy the attention of associations like the one I have the honor to address.

IV. What Shall be the Mathematics of Our Technical Schools?

A fourth problem that is thrust upon us by modern conditions relates to the mathematics of our technical schools. While we do not hear such vociferous assertions as we did a short time ago about the fact that "the doctrine of formal discipline has been exploded"—a beautiful catch phrase that was quite fashionable for a time—it is nevertheless rather axiomatic that the mathematics of a technical school should aim at something in addition to general culture or power. In a school of mechanics we need the mathematics of mechanics, and so for other special fields. And yet, as we look over courses of study for our agricultural schools we find only arithmetic in one college and mathematics through the calculus in another, while in the high schools the confusion is equally apparent. And what is true for agricultural schools is quite as true for other technical institutions. We have not even begun to think seriously about solving the problem for our high schools, and the same thing can be said for our industrial schools of a more elementary character. Mere technical mathematics alone has never succeeded, and the nature of the general mathematics that is best suited to develop power to handle the problems that confront the foreman who is erecting a skyscraper has never yet been determined. Here, then, is another serious problem that meets us when we try to settle upon the best course in mathematics for the technical schools of our country. These schools are hardly started in America as yet, although in Europe they are well established, and if we may judge from world experience their success is to depend in no small degree upon the quality of mathematics that shall enter into their curricula.

V. The Problem of Our Backward American Mathematics.

There were in the mind of those who initiated the work of the International Commission on the Teaching of Mathematics three

large problems. The first was that of letting each nation take stock of its own work in presenting mathematics to its youth, that it might have before it a kind of moving picture of its teaching, from the kindergarten through the university. The second was that of informing other nations of its work, to the end that all might profit by the success and failure of each. And the third problem was that of looking beyond the confines of one's own land and seeing what the rest of the world is doing. In the United States upwards of a dozen reports were issued, telling the story of our own work. But there remains the third problem, that of looking abroad and seeing wherein other nations are surpassing us, and then of finding the causes and the remedy. This is the problem of the immediate future, and it is proper on this occasion to set forth one of its phases.

When we compare, year for year, the work in mathematics here and abroad, we are struck by the fact that we in the United States are not only not the leaders, but in nearly every case we are distinctly behind the other prominent countries of the world. At the end of our seventh grade we are about a year behind, and at the end of our twelfth school year we are about two years behind other great educational nations in the teaching of mathematics. You say this to a professional pedagogue and he will make all sorts of apologies. He will say: "Oh, this is America, and we have different problems,"—as if that were any excuse. He may lay it to climatic conditions, to the necessity for assimilating a million immigrants a year, to the paucity of teachers in a new country, to the brief tenure of office of women teachers and even of men, to our shifting population, to our democracy of education, to the greater breadth of our curriculum, or to any one of dozens of other causes. But he is a rare educator who will come out and assert that we have a soft pedagogy that often dominates our elementary school, a sweet but mushy pedagogy that brings a maximum of temporary pleasure with a minimum of intellectual attainment. Perhaps the reason that he does not say this is that it is not true. But when one sees the vigor, contentment, good spirit, and comradeship that is found in this generation in so many of the schools that represent the influence of Pestalozzi in certain of the countries abroad, and then sees the intellectual progress that these schools foster, he must ques-

tion, if he is open-minded, the wisdom of those who have directed the American policy. We are behind; bad as the European arithmetic is, ours is worse; backward as the European boy may be in his algebra, ours is more so; faulty as may be his attainment in geometry, that of our boy or girl is still more so. I know of plenty of schools in which boys at the end of the twelfth school year have a good working knowledge of trigonometry, a fair command of the basal principles of analytics, and enough ability in the calculus to meet the demands of a pretty good course in analytic mechanics,—but they are not American schools. I join you in excusing ourselves; I know the standard explanations by heart; I am even willing to voice with every true American, of the type satirized by our foreign friends, in asserting our claim to having the longest river, the biggest lake, the tallest skyscraper, the wealthiest men, and the most abject poverty to be found anywhere. But after all this boasting is over, I have to confess to myself that in the teaching of the science in which this Association and I are interested America is behind, definitely and unquestionably behind, and I seek not for a dozen causes so much as for one good remedy.

When we attempt to free ourselves from our insular habit, and turn our attention to learning from the experience of the rest of the world instead of from the theories of the lecture room, we find that the work of the first two years is generally more definite in other countries than in ours. The idea that arithmetic shall be "merely incidental" in these years is not held abroad, and, indeed, has little scientific standing even here, although it has its advocates. Six years are elsewhere generally deemed sufficient to cover the essentials of arithmetic, the subject thereafter being reviewed and applied along with algebra and geometry. Instead of beginning formal algebra as a new topic in the ninth school year as with us, the subject is introduced by easy steps in grades VI and VII, so that the pupil is initiated by slow degrees into the advanced stage. Instead of holding geometry until the tenth school year, and then suddenly springing the demonstrative phase upon the pupil, this subject is introduced along with algebra in the elementary grades. In other words, instead of devoting grades VII and VIII to a business arithmetic that is often too difficult for the children, a

simple initiation is given to the algebra of the formula and equation, to geometric drawing and to the simpler demonstrations, and to such arithmetic as is within the grasp of the pupils of these grades. There is thus an intelligent preparation for formal algebra and demonstrative geometry that is quite lacking with us. Moreover, in the eleventh and twelfth school years there are often opportunities to take courses in trigonometry, analytics, the calculus, and mechanics, and sometimes mathematical astronomy, that are almost never found in our schools, and that have proved their worth by the results attained.

It would be a sad error if we should conclude from such a statement that our work is all bad and the foreign work all good. The human tendency that Horace satirized, of looking on our neighbor's possessions as better than our own, must always be recognized. The fact is that we have in America an excellent course in algebra and geometry, better in some respects than those found abroad. Our arithmetic work, too, has many features of superiority. But our deficiency seems to lie in three features: our dawdling work in early primary arithmetic, our neglect of the initial stage of algebra and geometry in grades VII and VIII, and our failure to offer advanced electives in the last two years of our high school.

I am well aware of the difficulties to be met in applying the remedy, and it is unnecessary to dwell upon them here. For example, I know the gap between democracy and aristocracy in education, but I don't believe it is as wide as many people think. I wish to set the problem before you rather than to attempt to point out the slow steps by which the solution may be effected. If an ideal is kept before our people we will all gradually move towards it, and this gradual trend is wiser and safer than any attempt suddenly to attain results that seem to us desirable. The problem is to preserve the serious, orderly mathematics that we have, while adopting such good features as the rest of the world may suggest to us.

VI. The Question of Parallelism.

A glance at the foreign schools, such as has been given, suggests another problem that we are rather certain to meet in the near future, the one of rearranging our curriculum. America

is one of the few countries in which algebra and geometry are commonly taught in tandem fashion. The required work in most of our high schools is one year of algebra followed by one year of geometry, and this followed by an elective course in algebra, and this by one in solid geometry. The rest of the world in general does not pursue such a plan. It carries its algebra and geometry separately, as we do, but it carries them side by side, say from grade VII through our high-school period. It may definitely assign two days of the week to algebra, two to geometry, and one to arithmetic; or it may make some other arrangement, but in any case the arrangement is based upon the idea of parallelism. I do not believe that America is ready for this at the present time. Indeed, I think that it never will be ready unless it adopts the plan of a six-year high school, beginning with grade VII or goes even further in following the European arrangement. But there are many arguments in favor of the scheme in case such an administrative change is made. At any rate the plan succeeds everywhere else, and because it does not succeed under our present conditions is no reason for believing that we shall not have to meet the problem in the changing conditions that are likely to be met in the near future.

VII. Are We Making Mathematics Interesting?

Among all of the experiments that have been made in teaching mathematics in this country, not much that is strikingly new has been evolved, nor much that seems permanent. We have had in general education a great many bubbles to prick, as witness the failure of the type of manual training that was a few years ago asserted to be a panacea, the fading away of the doctrines of concentration and correlation, the oblivion into which the culture epoch theory has passed, and the fate of the Grube method. To be sure, the Montessori system is now being skilfully exploited, but that, too, seems destined soon to be forgotten. Likewise in the teaching of mathematics we have our bubbles, bubbles blown with the enthusiasm of youth and pricked with the experience of years. Sometimes the bubble is a geometry syllabus, sometimes an impossible fusion of mathematical topics as far apart as Latin and chemistry, sometimes a "ratio

method," sometimes a wild adoption of the graph, and sometimes "practical mathematics." And hence it is not without hesitancy that I suggest what very likely may develop into another bubble, the problem of making mathematics more interesting in and for itself.

It is a fact that, although no two sets of educational statistics ever seem to prove quite the same thing, all statistics that touch upon the subject seem to show that mathematics ranks well up in the scale of pupils' interests. The latest set that I have happened to see placed it third or fourth from the top on a scale of twelve, with the vocational training, that was to have been our salvation, way down at the bottom of the whole list. However taught, mathematics always has in it the game element. You play the game, and you win if you really set about to do so, and when you win you have a definite result. We start, then, in the teaching of mathematics, with this great advantage.

But the question arises, are we making enough of this matter of interest? If we give a couple of pages of dull, dry, algebraic formalism, with nothing contributed by the teacher to enliven it, are we doing our best? If not, what more can we do?

The problem is not one to be solved in a moment. But let me suggest that the history of mathematics is not being used as wisely as we might use it in our classes. I do not think much of it as it appears in the form of notes in a text-book, but as outside material to be brought into class, to be the subject of a moment's inspiring talk by the teacher,—this is where its value seems to lie. And so it is with the recreations of mathematics, a subject on which we have considerable available literature. Are we using this material as we should? Is it feasible in our schools to establish mathematical clubs, such as Mr. Newhall describes in his monograph in the Commission report on mathematics in the secondary schools? I have myself ventured to suggest in a recent number of the *Teachers College Record,* under the title "Number Games and Number Rhymes," a few possibilities. But certain it is that there is an opportunity for serious work here, and that some good may be accomplished if we do not go to the extreme of making a mere bubble out of the effort. There would not be so much heard about the immediately practical in mathematics if we would show our pupils the

interest in the subject *per se,* the meaning of the subject in the larger life about us, and the comparative value that it has. I wonder if we ourselves ever stop to think of the effect of blotting out of existence every book or manuscript, say even on so important a subject as the general science of education, and also every book or manuscript on mathematics. In the former case the schools would open next week as they opened this; the teaching would go on as before; the world would know no difference save in a few institutions for the training of teachers, and even they might conceivably be the better off. But in the case of mathematics every great engineering project in the world would be stayed; the skyscraper would not be planned; the next ocean leviathan of steel could not be begun; the banking of the country would halt; all navigation would cease,—at least all safe navigation; the science of artillery would need to be begun anew; astronomy would stand aghast; mechanics would have again to frame its laws, and civilization would send out the hurry call for intellects to repair the damage. The question, then, as to what would happen if mathematics were taken away, might well be suggested to a mathematics club in a high school, and this is a type of dozens of other questions that would probably add to the interest that the pupils take in the study. Imagine the joy of a healthy-minded pupil when he first reads "Flatland, or Another World," or Hill's "Geometry and Faith," or when he comes to know Ball's "Mathematical Recreations" or White's "Scrap Book of Mathematics!" And so, at the risk of starting a new bubble, I suggest as one of the problems of our time the effort to make mathematics more interesting, independent of its narrow field of immediate applications.

VIII. The Function Problem.

It is perhaps well to inject into this discussion a problem that is a little more mathematical than the ones that have been suggested. For this purpose I select one of which a great deal has been said in Europe during the past five or ten years, and one of which we are soon to hear. I refer to the introduction of the function concept early in our work in mathematics, making of it a kind of unifying principle of the elementary science. I have not the time to refer to its recent history, to the rise of the

idea of thus using the concept in the efforts of the French engineers to improve their work; to its elaboration by men like Tannery; to its cool initial reception by the French teachers; to the subsequent efforts of Klein, and to its recent reception in Germany. Suffice it to say that there seems good reason for bringing before the pupil, as soon as he begins the study of algebraic symbols, the idea of function. This idea naturally enters into the study of the simplest formula, it is the essence of the equation, it is the major part of mensuration and trigonometry, and in the mathematics that follows it plays a part of ever-increasing importance. The question is, What shall be its fate in America? Shall we receive it with western enthusiasm, exploit it as we did graphs, go to an extreme from which we must recede as we did in that case, and finally, after much waste of energy, come down to a sane use of this valuable aid to the study of mathematics? Or shall we ask ourselves the question as to why we study the subject, what we propose to accomplish by it, where we can really use it to advantage, and what are the extremes to be avoided? If we take the latter course we shall reach the sane stage much more quickly than by the former route. Already the danger has appeared, and so it is well that we consider this as one of the problems demanding our serious attention.

IX. The Problem of Our Duty.

Addresses like the present one are ephemeral; they reach no conclusion, and they are not likely to provoke very serious thought. And yet they serve a purpose if they select from the wide range of questions of the day a few that seem to demand special attention. As I said at the outset, I have no definite solutions of any of these problems, or if I have I do not believe that much will be gained by laying them before you. The problem is the thing; the solution will follow in the general agreement of the large, silent, thoughtful body of American teachers. But it would not be right that I should close merely with the enunciation of a few questions, accompanied by remarks that are more or less rambling. I therefore wish to suggest one more problem, that of our present duty, and to indicate what seems to me its solution.

I believe it to be our duty to stand solidly against the lowering of the standard of mathematics that shall make of it only a science that is immediately instead of potentially practical. I believe that we should open the door of the great field of algebra and of geometry to every boy and every girl in our country. If they fail, let them substitute some other science for this, but offer them the chance. If we can extend our high school downwards to include grade VII the offer may be made in grades VII and VIII, and this will be better than to require the algebra and geometry of to-day. But to close the door of opportunity, in any one of the great branches of knowledge, is a crime, whether it be in biological science, in music, in hygiene, in language, in mathematics, in history, or in any other field of world importance. I also believe it to be our duty to favor the extension of high school mathematics downwards to grade VII at least, taught by the high school teachers, which teachers shall in the near future be trained in physics as well as our own science. With this should come the extension of electives upwards, to include courses in analytics, calculus, and mechanics. To say that this is impossible is to say that the American youth and the American teacher lack the abilities of their foreign colleagues. I believe it to be our duty to encourage in every way a proper industrial education, but to insist that serious, thoughtful mathematics is to have its place, and not become merely a collection of a few rules of thumb. I feel that we should guard against making the function concept ridiculous as we, in some quarters, made the graph an absurdity; but that we should accept it for what it is really worth and use it all through our teaching where, to use the jargon of the pedagogue, it "functions." I conceive, furthermore, that an association like this is unanimous in the belief that we should "hold fast to that which is good," not allowing ourselves to give up the serious study of algebra because it has not always been well taught in the past, nor the serious study of demonstrative geometry because someone has, for the millionth time, slaughtered Euclid. The possible improvements that are suggested by the problems that I have ventured to lay before you may be effected with only a natural and gradual change of the science as we have it; they demand no cataclysm, least of all the cataclysm that

would follow if some of our reformers had their way. When we see a man claiming that he speaks for the 95 per cent of high-school pupils in demanding the abolition of real mathematics, we will do well to listen to his objections, trying to remove any just cause for his complaints. But if we find that he is merely the advocate of the non-intellectual, and if he would take from us all the idealism that we have, then we have a right to put forward the claims of the boy and girl who really wish to learn and in whose souls idealism is beginning to take a start. But even more than this, we have the duty to do our best to bring this same spirit into the schools for the 95 per cent of whom we hear so much, and this I believe we shall do effectively only when our high school work begins earlier than at present, and we put some propaedeutic and practical work in grades VII and VIII.

And above all, it seems to me to be our duty to stand for the interest of mathematics for its own sake, for setting forth its beauty of symmetry, for voicing its poetry, for living its religion, and for exalting it for the truth that is sets forth so clearly and for the invariant properties that characterize it in every branch. It is only by being imbued with such feelings and ambitions that we can bring our pupils to love the subject and to feel the great mental uplift that comes from its study. May the statement of the few problems that I have set before you assist us all to maintain this spirit in the future as I am sure we have tried to maintain it in the past.

TEACHERS COLLEGE,
NEW YORK.

IN WHAT OPERATIONS IN ARITHMETIC SHOULD A PUPIL BE REQUIRED TO UNDERSTAND THE REASONS FOR THE STEPS TAKEN? IN WHAT OPERATIONS SHOULD SUCH A REQUIREMENT BE POSTPONED TO A MATURER AGE?

By A. M. Curtis.

Our subject is in the form of a question demanding an answer. *No definite answer can ever be given in the form of a rule to be followed in teaching children.* Children differ so widely in natural aptitude and in rates of development that work adapted to one third of a grade of pupils is not suitable for the remainder of the grade.

It is for this reason that I claim that no definite rule can obtain and that all is finally dependent on the teacher's judgment brought to bear on the needs of the individuals of a class regardless of "grade."

I am, therefore, driven away from my ideals when I begin to discuss our question from that most unsatisfactory platform—the best for the average.

It is one thing to ask a pupil to *do* in arithmetic. It is quite another thing to ask for an expression of the underlying argument which gives him his rule for the performance of his process.

The *complete* understanding of the four processes with integers is beyond a child's comprehension. And it is not necessary that he know the whole theory of the work in order to glean for himself what he does need—namely, speed and accuracy in the doing. With fractions, the case is different. The idea of fractions brings the need of illustration that the pupil may comprehend the language of the subject, and grow in appreciation of the ratio concept and the different units which fractions involve. Here the reasoning element is brought into prominence through an appeal, through the senses, to the proportion in things, the sizes of units. It is only by continued exercises of such nature that the abstraction takes lodgment.

The changes in fractions known as reductions, the necessity for like units in adding, and the like, are cases in point.

Decimal fractions and their relations to common fractions and per cents are not taught in a day. One learns to know their true relation and significance by repeated use, and emphasis on proper *reading* and *habit of thinking* as their application broadens.

But here as elsewhere in mathematics the new idea is one of appreciation—not of truth alone, but of a shorthand, compact notation, wise and useful, yet not void of difficulties in its acquirement. Here pupils must know and know thoroughly. And here, too, the reasoning is best learned through repeated contact in varied application to reality: approximating, proving, expressing the ideas in definite forms in logical order, and then learning to jump the details and fly to the conclusion.

The position of the decimal in the product is illustrative of this period of arithmetic's growth.

To multiply 1201.5 by .015, for example. If pupils ***read, really read,*** their rule is made clear. They must say ***fifteen thousandths of.*** What does ***fifteen thousandths of*** mean? Answer the question and the product's "decimal point" is placed by $15 \times$ 1-1000 of 1201.5 or 15×1.2015.

Such analysis can not be forced on children. They come to an understanding of it slowly in a majority of cases. They can, however, all be taught to *do,* should be so taught, and at the same time carefully trained to *read deeply,* that is, with deepening appreciation of words' meanings.

It is not all *do,* nor is it formal demonstrative reasoning on the part of the pupil that will win the day for him—our unfortunate "average."

I like the following plan of treatment for every subject in arithmetic as suited to our classes in public schools:

Explain by genetic mode of teaching each step and process presented in very simple, brief form—so as to appeal to the eye in strikingly clear arrangements. Follow with much drill and practice. In a week, present the whole subject again; drill and practice. And again and again present it. Let every teacher in succeeding grades present it.

It is by such repetition only that the slowly developing pupil

gains his understanding. The precocious pupil had his due in the first round of explanation.

Such a plan gives *every* pupil a fair chance, makes the work show all pupils that rules do not come into existence in haphazard fashion, but are expressions of the working out of principles and related thought. It stimulates to clear thinking and yet does not lose sight of the practical phase—do accurately and speedily business arithmetic.

ONEONTA, N. Y.

TAKE JOY HOME.

Take joy home,
And make a place in thy great heart for her,
And give her time to grow, and cherish her,
Then will she come and oft will sing to thee,
When thou art working in the furrows; aye,
Or weeding in the sacred hour of dawn.
It is a comely fashion to be glad:
Joy is the grace we give to God.

—*Jean Ingelow*

WHAT MATHEMATICAL SUBJECTS SHOULD BE INCLUDED IN THE CURRICULUM OF THE SECONDARY SCHOOLS.*

By Charles L. McKeehan,
of the Philadelphia Bar.

The President of your Association has asked me to discuss this subject from the point of view of the professions. Surely a daring undertaking for one who knows nothing about mathematics except the simplest operations in integers and fractions, who knows as an abstract proposition that two and two make four, but who frequently finds that they make three or five to him when he adds them in a column of figures, and (to come closer to the point) who never did really understand the principles of mathematics, although he graduated from a famous preparatory school and escaped from the university of Pennsylvania with a bachelor's degree.

In a delightful little Scotch play now being presented in Philadelphia, there is a scene in which a Scottish lad of about fifteen years is seen in his father's home on Sunday morning wrestling with the Shorter Catechism, with expression ranging from puzzled, hazy wonder, through irritation, indignation and distraction to blank despair. He mumbles half aloud "purification, justification, sanctification," and then bursts out with "Faither, I can't understand it!" The stern old Scotchman flashes back "Who's expectin' ye to understand it? *Learn it!*" I do not mean to attribute any such unreasonable attitude to the present day teachers of mathematics, and yet I believe that this scene as a whole brings back to most men a vivid memory of their boyhood struggles with mathematics.

(*To be continued*)

* Paper read before the Association of Teachers of Mathematics in the Middle States and Maryland.

NEW BOOKS.

New Analytic Geometry. By PERCY F. SMITH and ARTHUR SULLIVAN GALE. Boston: Ginn and Company. Pp. 342. $

The authors of this book have recognized the importance of "setting up" and studying functions by their graphs. The student is carefully led through the subject, being, for the most part, given just sufficient aid to make his own way. He is taught analytic methods and to formulate his own rules.

Durell's Arithmetics. Elementary. By FLETCHER DURELL and ELIZABETH HALL. Pp. 354. 48 cents. **Advanced.** By FLETCHER DURELL. Pp. 458. 64 cents. New York: Charles E. Merrill Company.

These two books taken together make an extensive treatment of over 800 pages and contain a great profusion of applications of number to concrete things in the various industries and departments of study. Effort has not only been made to apply number to things but to give the student a grasp of the processes. The weakest point would seem to be the definitions, and here is where many if not most arithmetics are at fault.

Intercollegiate Debates. Volume II. Edited by EGBERT RAY NICHOLS. New York: Hinds, Noble and Eldredge. Pp. 832. $2.00.

This book might as well have been named "Inter-High School Debates" for the language is simple and the arguments are clearly explained. Six of the discussions elaborate the brief treatment of the same questions in the earlier book, especially along the line of the points that have assumed a different complexion in contact with public controversy. The questions considered are of present-day interest and importance and the book is one that everyone interested in debates will want to read.

Old Paris. Its Social, Historical and Literary Associations. By HENRY C. SHELLEY. Boston: L. C. Page & Company. Pp. 354. $3.00.

The author is not giving in this volume a description of modern Paris, but, as the title indicates, of old Paris, for the inns, pleasure gardens, theaters, etc., described and illustrated have all been demolished to give way to modern Paris.

The city has for long years been one of the most attractive of the European capitals to the traveler and pleasure seeker, and the secret of the charm which captivates all visitors was according to Walpole that the Parisian lives in "Perpetual Opera" and "persists in being young when he is old." The life there is intensely social and hence the inn,

shop, theater, club, etc., have played a large part in the life of the city, all of which is described by the author in a very pleasing manner and with a profusion of excellent illustrations. In the descriptions of the salons which were somewhat peculiar to Paris much attention is given to the literary associations.

The Raphael Book. By Frank Roy Fraprie. Boston: L. C. Page & Company. Pp. 352. $2.50.

Perhaps there is no more lovable character in the history of art than Raphael and the reason for this undoubtedly lies in the fact that while he lived at a time when morals were low he never put on canvas anything but what represented the tender, gracious and beautiful. His pictures portray such emotions as mother love, divine aspiration, chaste and lovely themes.

This volume is an account of his life and place in the development of art, together with a description of his paintings and frescos. It is a book that is splendidly illustrated and will be read with pleasure and profit.

Industrial Mathematics. By Horace Wilmer Marsh. New York: John Wiley and Sons. Pp. 477. $2.00 net.

The aim of the author of this book is to make the student going into manufacturing shops proficient in the various mathematical operations by employing them in computation with technical formulas using actual commercial data.

Every chapter in this book presents the fundamental principles of the chapter topic, illustrated by numerous examples and applied in many industrial problems with actual commercial data.

The problems are of four kinds:

(1) Problems with complete data,

(2) Problems requiring measurement and computation of models,

(3) Problems requiring measurement and computation of materials and machines in shops and laboratories,

(4) Occasional problems with data incomplete, so that students may appreciate real conditions.

If you wish a text-book covering the fundamentals of arithmetic, industrial computation, and the simplest elements of algebra and trigonometry, you will select this book.

It contains a great wealth of material.

A Source Book of Problems for Geometry. By Mabel Sykes. Boston: Allyn and Bacon. Pp. 372..

The author of this book considers that one great object in the study of geometry is to lead to an appreciation of form and one who works through this book carefully will gain much in that respect. For students of design it would prove a very valuable study.

On the Foundations and Technic of Arithmetic. By GEORGE BRUCE HALSTEAD. Chicago: Open Court Publishing Co. Pp. 133. $1.00.

This book gives an account of the origin, foundations, meaning and aim of arithmetic, and teachers of that subject will find it interesting and profitable reading.

Non-Euclidean Geometry. By ROBERTO BONOLA. Translated into English by H. S. CARSLAW. Chicago: Open Court Company. Pp. 268. $2.00.

This work is a critical and historical study of the development of the subject and one that will be welcomed by teachers of geometry who desire to know more of this most interesting field. The treatment is clear and concise and there are a large number of references which add to the value of the book.

To Jerusalem Through the Land of Islam. By MADAME HYACINTHE LOYSON. Chicago: Open Court Publishing Co. Pp. 325. $2.50 net.

The disturbed conditions in the Orient today make this a very timely volume. The author's sympathetic viewpoint enables her to see much that was good and noble in the Jews and Moslems which would have escaped the prejudiced traveler. It is not only interesting reading but is full of information about these peoples. The illustrations are numerous and good.

Extemporaneous Speaking. By PAUL M. PEARSON and PHILIP M. HICKS. New York: Hinds Noble and Eldridge. Pp. 268. $1.25.

"Leadership is the reward of the man who possesses the power of effective speech," and this book is a working text for accomplishing this end. The first part (58 pages) is devoted to the elements and principles and the remainder of the book to a wide range of speeches for study. A careful perusal of its pages will certainly help those who desire to speak with force and effectiveness.

Radioactive Substances and their Radiations. By E. RUTHERFORD. Cambridge: University Press; G. P. Putnam's Sons, American representatives. Pp. 699. $4.50 net.

This is not a new edition of the author's earlier work on radioactivity, but a new volume giving an accurate and concise account of the whole subject as it is known today. Those who desire the latest word in this field will find it given here by one of the most active investigators and authorities in the world on the subject.

Studying the Short-Story. By J. BERG ESENWEIN. New York: Hinds Noble and Eldridge. Pp. 438. $1.25 net.

The tendency of the age is to find out scientifically all that may be known about all things, and it is no matter for surprise that the short-

story, the most widely read literary form today, should at last be brought into the laboratory and subjected to exhaustive analysis. It is remarkable, however, how entertaining such a penetrating study can be made, as is shown in Studying the Short-Story, a new work from the literary laboratory of Dr. Berg Esenwein, the Editor of Lippincott's Magazine.

Sixteen stories are transplanted bodily for this experiment course.

The most striking feature of the book is this: In each group the short-stories are printed with wide margins. The first story has all its secrets worked out by the editor. The open pages of the second story invite the aspiring pencil to do as well in the way of analysis and notes!

Each of the eight sections contains also references for further reading, a list of ten representative stories of that particular type, together with the names of the books that contain them, and a series of stimulating questions which serve to quicken the observation of the reader.

A Text-book of Mathematics and Mechanics. By CHARLES A. A. CAPITO. London: Charles Griffin & Company; Philadelphia: J. B. Lippincott Co. Pp. 398. $4.00 net.

This book has been written primarily for students in technical and engineering courses and covers analytical geometry, calculus and mechanics. The analytical geometry covers sixty four pages and treats of the straight line, circle, parabola, ellipse, and hyperbola separately and then shows that they all belong to the one category—the conic section. The calculus part covers one hundred and five pages and treats of the usual topics in a course for engineers. The mechanics part includes a rather comprehensive course not omitting vectors, hydromechanics, pneumatics, etc.

It is interesting to see how soon the author gets at the essentials for the technical and engineering student. No time or space is wasted on side issues and the whole ground covered keeps steadily to the requirement of usefulness to the student. The author seems to have produced a very clear and carefully written work.

NOTES AND NEWS.

THE tenth annual meeting of The Association of Mathematical Teachers in New England was held at the Massachusetts Institute of Technology in Boston on December 7, 1912.

The meeting was called to order by President Galbraith at 10:30 A.M. and the first business was the election of officers for 1913. The nominating committee reported the following list which was unanimously elected. President: William B. Carpenter, Mechanic Arts H. S., Boston; Vice President: Prof. Wm. A. Moody, Bowdoin College, Brunswick, Me.; Treasurer: F. W. Gentleman, Mechanic Arts H. S., Boston; Members of the Council: Miss Sarah J. Bullock, High School, Arlington, Mass.; Edwin A. Shaw, Prin. High School, Natic Mass.

Upon recommendation by the council, the association voted that:

(1) "This association adopts the *Mathematics Teacher* as its official organ, provided that magazine will print full reports of our meetings, and will make the subscription price 50 cents per year to members of this Association: this subscription to be included in our present annual dues."

(2) "The President and Secretary shall constitute a committee to confer with the Middle States and Maryland Association in regard to co-operation and joint meetings."

An extended discussion of methods of teaching algebra and geometry was introduced by two short talks, one on the "Stripe" in Geometry by Mr. John A. Marsh, of the English High School, and the other on "How I Teach Factoring," by Mr. Wm. B. Carpenter, of the Mechanic Arts H. S.

Following the morning session lunch was served in the Technology Union. The intermission lasted until 2 P.M.

The afternoon session was held in Huntington Hall. Prof. David Eugene Smith, of Columbia University, delivered a most timely address upon the problems of secondary mathematics. After this address Prof. H. W. Tyler, of the Massachusetts Institute of Technology, exhibited a selection of lantern slides upon the history of the mathematical sciences.

Prof .Smith's address is included as a part of these minutes, by action of the council, taken at their meeting on January 25.

The Council for 1913.

William B. Carpenter, *President,* Mechanic Arts High School, Boston, Mass.; Professor William A. Moody, *Vice-President,* Bowdoin College; Harry D. Gaylord, *Secretary,* 98 Hemenway St., Boston; F. W. Gentleman, *Treasurer,* Mechanic Arts High School, Boston; Professor Frederick S. Woods,† Massachusetts Institute of Technology; Miss A. Laura Batt,* High School, Somerville, Mass.; Professor Julian L. Coolidge,* Harvard University; Ernest G. Hapgood,† Girls' Latin School, Boston; Edwin A. Shaw,‡ High School, Natic, Mass.; Miss Sarah J. Bullock,‡ High School, Arlington, Mass.

THE Pittsburgh Section of The Association of Teachers of Mathematics in the Middle States and Maryland held their fall meeting in the Teachers' Room., Carnegie Library, at 10 A. M., Saturday, November 9, with the following program: "Some Ideas on the Study of Geometry," Charles R. Schultz, California Normal School; "The Opportunities the University Offers to Teachers of Mathematics," F. J. Holder, University of Pittsburgh; "The Service of the Section to its Members," W. F. Long, Central High School; open discussion of the question: How Can the Ends of the Study of Geometry be Best Attained?

THE United States Bureau of Education has just published a "Bibliography of the Teaching of Mathematics" covering the period from 1900 to 1912, by David Eugene Smith and Charles Goldziher. This bulletin gives 1849 titles of books and articles on the teaching of mathematics that have appeared since 1900. The bulletin will be sent gratis upon application to the United States Commissioner of Education, Washington, D. C.

NEW MEMBERS.

JACKSON, HARVEY W., 703 Washington Ave., Dunkirk, N. Y.
AIKEN, CARRIE E., 111 E. Fifth St., Jamestown, N. Y.
PARTRIDGE, ALSA, 61 Ketchum Place, Buffalo, N. Y.

* Term expires 1913.
† Term expires 1914.
‡ Term expires 1915.

RAMLER, OTTO J., 338 Franklin St., Buffalo, N. Y.
BALL, JENNIE, 517 S. Academy St., Medina, N. Y.
CROW, MARTHA W., York and Memphis Sts., Philadelphia, Pa.
EATON, CLARA C., 421 E. 88th St., New York City.
MERRING, MERTON D., Oxford, N. Y.
WHITE, HOMER ORSON, 145 Walnut St., Corning, N. Y.
SIMONS, EDITH, Newfoundland, Wayne Co., Pa.
SHIELDS, EMILY L., The Baldwin School, Bryn Mawr, Pa.
AIDAN, REV. BROTHER C. S. C., Notre Dame, Ind.
BECK, W. E., 1219 Nebraska St., Sioux City, Ia.

Home Progress is a monthly magazine now in the second volume published by Houghton, Mifflin Company and devoted to the enrichment of family life. It contains articles on home nature study, suggestions for keeping healthy, handicraft work, and many other things of interest and profit to the home.

Good Health Magazine is the organ of the Health and Efficiency League of America and contains much in the way of valuable hints for the care of the health. It is published monthly at Battle Creek.

For a daily newspaper which is clean and gives the news of the world in a clear, concise, and readable form there seems to be none which excels the *Christian Science Monitor* published
be none which excells the *Christian Science Monitor* published in Boston. It is not a paper devoted to the spread of the doctrine of Christian Science but one devoted to clean journalism.